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THE HIGHWAY SPIRAL FOR COMBINING CURVES OF DIFFERENT RADII

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HIGHWAY DIVISION

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THE HIGHWAY SPIRAL FOR COMBINING CURVES OF DIFFERENT RADII

Paul Hartman,[1] M. ASCE

SYNOPSIS

The highway spiral connecting a tangent and a circular arc is rigorously defined mathematically. When two circular arcs must be connected by a spiral, mathematical exactness gives way to the approximations of the theory of the osculating circle. The theory works well for flat spirals but it is not satisfactory for sharper spirals.

This paper derives rigorous equations for the highway spiral connecting circular arcs of different radii, which show the extent of the approximations involved in the theory of the osculating circle. Three graphs based on these equations permit rapid and accurate determination of the corrections to be applied to values obtained from the theory. The engineer with a working knowledge of the theory should find the determination and application of these corrections straightforward and simple.

The derivations are cumbersome and extensive. They are given in outline form only but in sufficient detail to permit checking.

INTRODUCTION

The spiral connecting two circular arcs is a portion of the spiral which would be used to connect the sharper arc to a tangent. Thus, a 200-foot spiral connecting a 4° and a 6° circular arc is the last third of a 600-foot spiral running from a tangent to a 6° arc. To distinguish between the 200-foot spiral and the 600-foot spiral of which it is a part, the former will be called a compound spiral while the latter will be called a simple spiral.

A simple spiral is shown in Fig. 1. The spiral is arc APB connecting tangent HE with circular arc BG. The circular arc is extended backwards to C where it is parallel to HE. The offset of the arc, produced, from the tangent, DC, is designated by the letter p in keeping with Barnett's[2] designations which will be used throughout this paper. The deflection angle to any point, P, (angle EAP) is ϕ. The central angle of the spiral to P is θ and the full central angle is θ_s.

A compound spiral is shown in Fig. 2a. The spiral is arc APB connecting circular arcs GA and BH. The sharper arc, GA, with radius OA, is extended forward to C on the common radial line O'D. The flatter arc, BH, with radius O'B, is extended backwards to D. The shift, CD, is designated p_a. The central angles of arcs AC and DB are designated β and α, respectively, a notation which is not in keeping with Barnett's notation for reasons that will be explained subsequently. The deflection angle EAP to any point, P, on the

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2. TRANSITION CURVES FOR HIGHWAYS, Joseph Barnett, PRA 1940.

spiral is ϕ_a. The angle between the tangent to the spiral at P and the main tangent, AE, is θ. (The tangent at P is not shown in the sketch).

Also

D_1 = degree of flatter circular arc, in degrees
D_2 = degree of sharper circular arc, in degrees
$D = D_2 - D_1$, or degree of curve for simple spiral
L_s = length of simple spiral, in feet
L_a = length of compound spiral, in feet

$$\Delta_1 = \frac{D_1 L_a}{200}$$

$$\Delta_2 = \frac{D_2 L_a}{200}$$

$$\theta_s = \frac{D L_s}{200}$$

$$\theta_a = \frac{(D_2 - D_1)\, L_a}{200} = \Delta_2 - \Delta_1 \text{ (nominal spiral angle)}$$

The theory of the osculating circle assumes that a spiral departs from an osculating circle in the same manner that an equivalent simple spiral departs from the tangent. For the simple spiral to be equivalent to the compound spiral, the spiral angle, θ_s, of the former must be equal to the nominal spiral angle, θ_a, of the latter and L_s must be equal to L_a.

Thus, in Fig. 2b, which is an enlargement of a portion of Fig. 2a, the deflection angle, ϕ_a, to any point P, distant ℓ (measured along the arc) from A, is the difference between two angles, the deflection angle to the osculating circle, EAQ, and angle PAQ. The latter is equal to ϕ, the deflection angle of an equivalent simple spiral to a point distant the same length ℓ from point A in Fig. 1. Expressed mathematically

$$\phi_a = \frac{D_2 \ell}{200} - \phi$$

For a set-up at B this equation would become

$$\phi_a = \frac{D_1 \ell}{200} + \phi$$

The length ℓ in each case is the distance along the spiral from the set-up point to the sighted point.

The theory of the osculating circle further assumes that the shift between the circular arcs produced, p_a, is equal to p of an equivalent simple spiral, and that β and α are equal to Δ_2 and Δ_1, respectively.

Corrections

The derivations show that all of the assumptions of the theory of the osculating circle are in error, the magnitude of the errors varying with Δ_1 and Δ_2. For flat spirals the errors are negligible but with increasing sharpness, the errors become so large that the theory can not be used without applying the corrections which are discussed below.

The derivation of β shows that it is always larger than Δ_2 by a quantity which will be designated C_b.

$$C_b = 0.03655\Delta_2\Theta_a(\Delta_2-\Theta_a)\left[1+121\left\{24\Delta_2(\Delta_2-\Theta_a) - 7\Theta_a^2\right\}10^{-8}\right] \tag{1}$$

where θ_a and Δ_2 are expressed in degrees and C_b is expressed in seconds. The expression is set up in terms θ_a and Δ_2 rather than Δ_1 and Δ_2 because the use of these terms simplifies a subsequent equation and it seems logical to use the same terms in all equations.

The correction C_b may be determined using the graph of Fig. 3 which consists of curves of C_b versus θ_a for various values of Δ_2. To illustrate the use of the graph the value of β will be determined for a 400-foot compound spiral connecting a 10° arc with a 20° arc. The arguments are

$$\Delta_2 = \frac{20 \cdot 400}{200} = 40°$$

and

$$\Theta_a = \frac{(20-10)\ 400}{200} = 20°$$

The value of C_b is the ordinate of the curve marked "Δ_2 = 40°" at the point where the abcissa, θ_a, is 20°. It is 596″ or 9′ 56″ and β is equal to 40° 09′ 56″. Since the complete central angle is $\Delta_1 + \Delta_2$, α must be equal to Δ_1 minus C_b or 19°50′04″.

The value of the shift, p_a, is always less than p by a quantity which will be designated C_p.

$$C_p = 0.222\, L_a\Delta_2\Theta_a(\Delta_2-\Theta_a)10^{-7} \tag{2}$$

where C_p and L_a are expressed in feet and Δ_2 and θ_a are expressed in degrees. In the graph of Fig. 4, curves of C_p for an L_a of one thousand feet versus θ_a are plotted for various values of Δ_2. Using the spiral of the preceding example, the value of C_p from the graph is 0.354. Multiplying by 400 and dividing by one thousand C_p is 0.142. The value of p from standard spiral tables for a θ_s of 20° is 11.584. The value of p_a is, therefore, 11.442.

The expression for the correction to be applied to intermediate deflection angles of the theory is cumbersome in its general form (see Eq. 16). The expression for the correction to the deflection angle to the end point of the spiral is simpler. It is

$$\begin{aligned} C_a &= 0.01218\Delta_2\Theta_a(\Delta_2-\Theta_a) + 0.0031\,\Theta_a^3 + 0.23\,\Theta_a^5 10^{-7} \\ &+ 0.353\,\Delta_2^3\Theta_a(\Delta_2-2\Theta_a)10^{-6} + 0.295\Delta_2\Theta_a^3(181\Delta_2-61\Theta_a)10^{-8} \end{aligned} \tag{3}$$

where C_a is in seconds and Δ_2 and θ_a are in degrees. Values of C_a may be obtained from the graph of Fig. 5 which consists of curves of C_a versus θ_a for various values of Δ_2. C_a may be determined from this graph with more precision than is ordinarily required for any practical layout.

It should be noted that the deflection angle to the simple spiral is assumed to be directly proportional to l^2, i.e., ϕ is equal to $\frac{\theta_s}{3}\left(\frac{l}{L_s}\right)^2$. Standard spiral deflection angle tables list angles computed from the formula

$$\phi = \frac{\theta_s}{3}\left(\frac{l}{L_s}\right)^2 - 0.31\ \theta_s^3\ 10^{-2} - 0.23\ \theta_s^5\ 10^{-7}$$

where ϕ and θ_s are expressed in degrees and the two negative terms are in seconds. The negative terms are usually small and so is C_a. By including these terms in C_a, two corrections which separately might be insignificant, together may have an appreciable value.

To obtain the correction to a deflection angle to an intermediate point on the spiral, it is necessary to assume that the spiral runs from the set-up point to the sighted point only. θ_a and Δ_2 must be computed for this abbreviated spiral and are then used as arguments to determine C_a either from the graph or from Eq. 3.

When running from the sharper to the flatter arc

$$\phi_a = \frac{D_2 l}{200} - \phi + C_a \tag{4}$$

and when running in the opposite direction

$$\phi_a = \frac{D_1 l}{200} + \phi - C_a \tag{5}$$

Computation of deflection angles for the spiral used in the preceding illustrative examples is shown in Table I. Set-up is assumed at the flatter end of the spiral, Sta. 10 + 00, and the deflection angles are computed to the tenth points.

The deflection angles to the osculating circle are listed in Col. 2. They are equal to $\frac{D_1 l}{200}$. Col. 3 contains the deflection angles of an equivalent simple spiral. ϕ is equal to $\frac{\theta_a}{3}\left(\frac{l}{L}\right)^2$ so the value of ϕ (for Sta. 10 + 40) is $\frac{20}{3}\ (0.1)^2$ which is equal to $0.^\circ 067$ or $4'$. Cols. 4, 5, 6, 7, and 8 contains values of D_2, D, L_a, Δ_2 and θ_a for the "abbreviated" spirals, spirals running from the set-up point to the sighted point only. Thus, for the deflection angle to the midpoint of the spiral (Sta. 12 + 00) the abbreviated spiral runs from a 10° arc to a 15° arc and is 200 feet long. D_2 is 15°, D is 5°, and L_a is 200 feet long. From these values Δ_2 and θ_a are determined to be 15° and 5°, respectively. Using these values as arguments the graph of Fig. 5 gives 21" as the value of C_a. Values of C_a are listed in Col. 9

The deflection angle given by the theory is the sum of Col. 2 and 3, which is listed in Col. 10. The correct deflection angle listed in Col. 11 is equal to Col. 10 minus Col. 9.

Deflection angles for the same spiral run in from the sharp end (Sta. 14 + 00) are computed in Table II. Notice that Cols. 3, 5, 6, and 8 are identical.

The corrections applied to the deflection angles of the first half of the spiral (the half nearer the point of set-up) are negligible. They are listed here to point up the fact that the best method of laying out a spiral may well

be to work from both ends toward the middle. Such a procedure would eliminate the need for computing C_a values and would also eliminate the long sights which are taken last in the field layout and are, therefore, most liable to error.

As a check on the precision of the corrections, the coordinates of the end point of a 200-foot spiral running from a 40° to a 20° curve were computed by the following methods:

1) theoretical equations for X and Y
2) traverse along the radii, using the value of β and p_a obtained from Eq. 1 and 2
3) traverse along the chords of a 10-point spiral

The theoretical coordinates are: X, 159.398; and Y, 105.088. The coordinates computed by the other methods did not differ from these values by more than 0.001.

The only correction that applies to a simple spiral is C_a, and that only when the set-up is at some point on the spiral other than the TS. For a set-up at the SC, D_2 is equal to the degree of the circular curve, D_1 is equal to the degree of the osculating circle at the sighted point, and L_a is equal to the difference in stationing of the ends of the line of sight. Similarly, for a set-up between TS and SC, the spiral between set-up and sighted point is treated as a compound spiral.

Computations of the deflection angles to the fifth points of a simple spiral is shown in Table III. The spiral is 200 feet long to a 30° curve. The transit set-up is assumed at Sta. 31 + 00, the SC. The columns are identical with those in Tables I and II.

Derivation

The degree of curvature of the highway spiral varies uniformly with length of curve. In Fig. 2a the degree of the spiral varies from D_2 at point A to D_1 at point B. The degree of curvature at any point, P, distant ℓ from point A, is

$$D_p = D_2 - (D_2 - D_1)\frac{\ell}{L_a} \tag{6}$$

Referring to Fig. 6 which shows an enlarge portion of Fig. 2a, the radius at any point, P, is R_p which is equal to $\frac{1}{D_p}$ if lengths are expressed in stations and angles in radians as they will be throughout the derivation. Then

$$d\ell = R_p d\theta = \frac{d\theta}{D_p} \tag{7}$$

Substituting for D_p its value from Eq. 6 and integrating

$$\theta = D_2\ell - (D_2 - D_1)\frac{\ell^2}{2L_a} \tag{8}$$

Letting $\frac{\ell}{L_a}$ equal K and substituting θ_a for $\frac{(D_2 - D_1)\,L_a}{2}$ and Δ_2 for $\frac{D_2 L_a}{2}$

$$\Theta = 2\Delta_2 K - \Theta_a K^2 \tag{9}$$

Also

$$dy = d\ell \ \sin \Theta \tag{10}$$

and

$$dx = d\ell \ \cos \Theta \tag{11}$$

Substituting for sine and cosine the respective series expansions in θ and integrating

$$y = \ell \left(\Delta_2 K - \frac{1}{3} \Theta_a K^2 - \frac{1}{3} \Delta_2^3 K^3 + \_\_\_\_ \right) \tag{12}$$

and

$$x = \ell \left(1 - \frac{2}{3} \Delta_2^2 K^2 + \frac{1}{2} \Delta_2 \Theta_a K^3 - \frac{1}{10} \Theta_a^2 K^4 + \_\_\_\_ \right) \tag{13}$$

Dividing y by x

$$\tan \phi_a = \Delta_2 K - \frac{1}{3} \Theta_a K^2 + \frac{1}{3} \Delta_2^3 K^3 - \_\_\_\_ \tag{14}$$

from which

$$\phi_a = \Delta_2 K - \frac{1}{3} \Theta_a K^2 + \frac{1}{90} \Delta_2^2 \Theta_a K^4 - \frac{1}{90} \Delta_2 \Theta_a^2 K^5 + \_\_\_ \tag{15}$$

Notice that the first term on the right is the deflection angle to the osculating circle and that the second term is ϕ. Thus these two terms are the deflection angle by osculating circle theory and the remaining terms comprise a correction, C_a, which is

$$C_a = \frac{1}{90} \Delta_2 \Theta_a K^4 (\Delta_2 - \Theta_a K) + \frac{8}{2835} \Theta_a^3 K^6 + \frac{32}{467,775} \Theta_a^5 K^{10}$$
$$+ \frac{1}{945} \Delta_2^3 \Theta_a K^6 (\Delta_2 - 2\Theta_a K) + \frac{1}{113.400} \Delta_2 \Theta_a^3 (181\Delta_2 - 61\Theta_a K) K^8 \tag{16}$$

Eq. 5 is the same as Eq. 16 except for units and the fact that K has been taken equal to unity in Eq. 5.

To obtain equations for β and p_a, the closed polygon shown by heavy lines in Fig. 7 is solved "by traverse". The direction of OA is assumed to be North and the sums of the latitudes and departures of the polygon sides are equated to zero.

$$\Sigma \text{Lat.} = O = R_2 - Y - R_1 \cos (2\Delta_2 - \Theta_a) + (R_1 - R_2 - p_a) \cos \beta \tag{17}$$

and

$$\sum \text{Dep.} = X - R_1 \sin(2\Delta_2 - \Theta_a) + (R_1 - R_2 - p_a)\sin\beta \tag{18}$$

Solving Eq. 17 and 18 for β

$$\cos\beta = \frac{Y + R_1\cos(2\Delta_2 - \Theta_a) - R_2}{R_1 - R_2 - p_a} \tag{19}$$

and

$$\sin\beta = \frac{R_1\sin(2\Delta_2 - \Theta_a) - X}{R_1 - R_2 - p_a} \tag{20}$$

Dividing Eq. 20 by Eq. 19 to eliminate p_a

$$\tan\beta = \frac{R_1\sin(2\Delta_2 - \Theta_a) - X}{Y + R_1\cos(2\Delta_2 - \Theta_a) - R_2} \tag{21}$$

All terms on the right may be expressed in terms of Δ, and θ_a. Division then yields

$$\tan\beta = \Delta_2 + \frac{1}{3}\Delta_2^3 + \frac{1}{30}\Delta_2^2\Theta_a - \frac{1}{30}\Delta_2\Theta_a^2 + \text{----} \tag{22}$$

from which

$$\beta = \Delta_2 + \frac{1}{30}\Delta_2^2\Theta_a - \frac{1}{30}\Delta_2\Theta_a^2 + \frac{1}{315}\Delta_2^4\Theta_a - \frac{2}{315}\Delta_2^3\Theta_a^2 + \frac{17}{7560}\Delta_2^2\Theta_a^3 + \frac{1}{1080}\Delta_2\Theta_a^4 \text{----} \tag{23}$$

In Eq. 23 Δ_2 is the nominal central angle and the remaining terms comprise a correction, C_b, which is added to Δ_2 to obtain β and subtracted from Δ_1 to obtain α.

With β known, either Eq. 19 or 20 may be used to solve for p_a. Using Eq. 20

$$p_a = \frac{(R_1 - R_2)\sin\beta - R_1\sin(2\Delta_2 - \Theta_a) + X}{\sin\beta} \tag{24}$$

Again expressing all terms on the right in terms of Δ_2 and θ_a, division yields

$$p_a = L_a\left[\frac{1}{12}\Theta_a - \frac{1}{240}\Delta_2^2\Theta_a + \frac{1}{240}\Delta_2\Theta_a^2 - \frac{1}{336}\Theta_a^3 + \frac{1}{10,080}\Delta_2^4\Theta_a - \frac{1}{5040}\Delta_2^3\Theta_a^2 + \frac{1}{18,900}\Delta_2^2\Theta_a^3 + \frac{1}{21,600}\Delta_2\Theta_a^4 + \frac{1}{15,840}\Theta_a^5 \text{----}\right] \tag{25}$$

But

$$p = L_s\left[\frac{1}{12}\Theta_a - \frac{1}{336}\Theta_a^3 + \frac{1}{15,840}\Theta_a^5 - \text{----}\right] \tag{26}$$

So

$$p_a = p - L_a\left[\frac{1}{240}\Delta_2\Theta_a(\Delta_2-\Theta_a) - \frac{1}{10,080}\Delta_2^4\Theta_a + \frac{1}{5040}\Delta_2^3\Theta_a^2\right.$$

$$\left. - \frac{1}{18,900}\Delta_2^2\Theta_a^3 - \frac{1}{21,600}\Delta_2\Theta_a^4 \text{----}\right] \tag{27}$$

Calling the bracketed term C_p and ignoring all terms within the bracket after the first term because they are negligible for any practical highway spiral,

$$C_p = \frac{1}{240}\Delta_2\Theta_a(\Delta_2-\Theta_a)\,L_a \tag{28}$$

where C_p is the correction to be subtracted from p_a.

CONCLUSION AND COMMENT

The derivations replace the theory of the osculating circle with mathematical exactness which permits the precise layout of a highway spiral between the branches of a compound curve, regardless of sharpness. The procedure is, in effect, an extension of osculating circle theory. The graphs of Figs. 3, 4 and 5 allow rapid and accurate determination of the corrections for any practical spiral although in cases where precise ties are desired, it would be preferable to compute C_b by means of Eq. 1.

Computations may be simplified and the precision of field layout increased if the spiral is laid out from both ends. Such a procedure eliminates the need for computing the corrections to deflection angles to the intermediate points of the spiral and also eliminates the long sights to the far end of the spiral which are a source of error in ordinary spiral layout.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Intermediate Values							
Sta.	Oscul. Circle	ϕ	D_2	D	L_a	Δ_2	Θ_a	C_a	Sum (2) + (3)	ϕ_a (10) - (9)
	° '	° '	°	°	ft.	°	°	"	° '	° ' "
14+00	20-00	6-40	20	10	400	40.0	20.0	225	26-40	26-36-15
+60	18-00	5-24	19	9	360	34.2	16.2	137	23-24	23-21-43
13+20	16-00	4-16	18	8	320	28.8	12.8	80	20-16	20-14-40
+80	14-00	3-16	17	7	280	23.8	9.8	42	17-16	17-15-18
+40	12-00	2-24	16	6	240	19.2	7.2	21	14-24	14-23-39
12+00	10-00	1-40	15	5	200	15.0	5.0	10	11-40	11-39-50
+60	8-00	1-04	14	4	160	11.2	3.2	4	9-04	9-03-56
11+20	6-00	0-36	13	3	120	7.8	1.8	1	6-36	6-35-59
+80	4-00	0-16	12	2	80	4.8	0.8	0	4-16	4-16-00
+40	2-00	0-04	11	1	40	2.2	0.2	0	2-04	2-04-00
10+00										

TABLE I

Deflection Angles for 400-foot Spiral with

$D_1 = 10°$ and $D_2 = 20°$. Set-up at Sta. 10+00 on 10° curve.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Intermediate Values							
Sta.	Oscul. Circle	ϕ	D_2	D	L_a	Δ_2	Θ_a	C_a	(2) - (3)	ϕ_a (9) + (10)
	° '	° '	°	°	ft.	°	°	"	° '	° ' "
10+00	40-00	6-40	20	10	400	40	20.0	225	33-20	33-23-45
+40	36-00	5-24	20	9	360	36	16.2	157	30-36	30-38-37
+80	32-00	4-16	20	8	320	32	12.8	104	27-44	27-45-44
11+20	28-00	3-16	20	7	280	28	9.8	65	24-44	24-45-05
+60	24-00	2-24	20	6	240	24	7.2	36	21-36	21-36-36
12+00	20-00	1-40	20	5	200	20	5.0	19	18-20	18-20-19
+40	16-00	1-04	20	4	160	16	3.2	8	14-56	14-56-08
+80	12-00	0-36	20	3	120	12	1.8	3	11-24	11-24-03
13+20	8-00	0-16	20	2	80	8	0.8	1	7-44	7-44-01
+60	4-00	0-04	20	1	40	4	0.2	0	3-56	3-56-00
14+00										

TABLE II

Deflection Angles for 400-foot Spiral with

$D_1 = 10°$ and $D_2 = 20°$. Set-up at Sta. 14+00 on 20° curve.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Intermediate Values							
Sta.	Oscul. Circle	ϕ	D_2	D	L_a	Δ_2	Θ_a	C_a	(2) - (3)	ϕ_a (9) + (10)
	° '	° '	°	°	ft.	°	°	"	° '	° ' "
33+00	30-00	10-00	30	30	200	30	30.0	84	20-00	20-01-24
+60	24-00	6-24	30	24	160	24	19.2	49	17-36	17-36-49
32+20	18-00	3-36	30	18	120	18	10.8	21	14-24	14-24-21
+80	12-00	1-36	30	12	80	12	4.8	6	10-24	10-24-06
+40	6-00	0-24	30	6	40	6	1.2	0	5-36	5-36-00
31+00										

TABLE III

Deflection Angles for a 200-foot Spiral Joining a Tangent and a 30° Curve. Set-up on the 30° Curve.

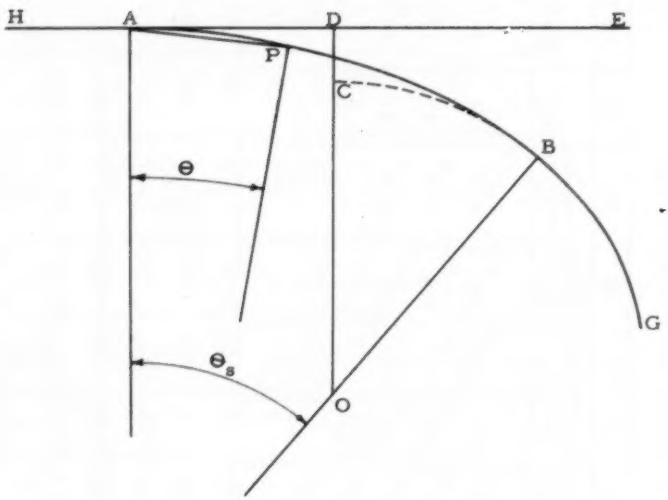
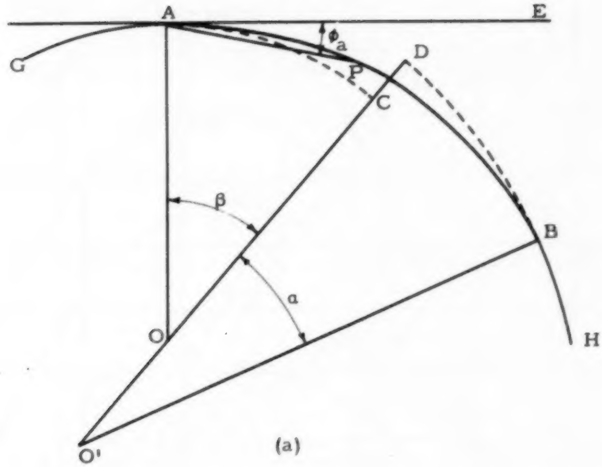
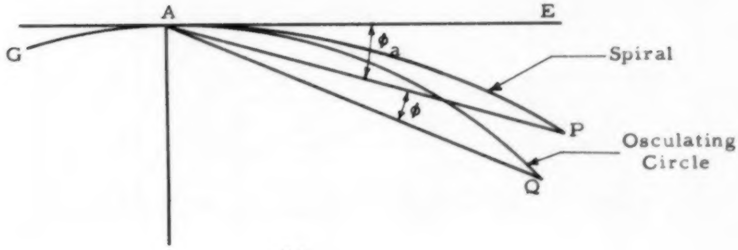


Fig. 1



A
E
G
ϕ_a
D
P
C
β
B
a
O
H
O'

(a)



A
E
G
ϕ_a
Spiral
ϕ
P
Osculating Circle
Q

(b)

Fig. 2

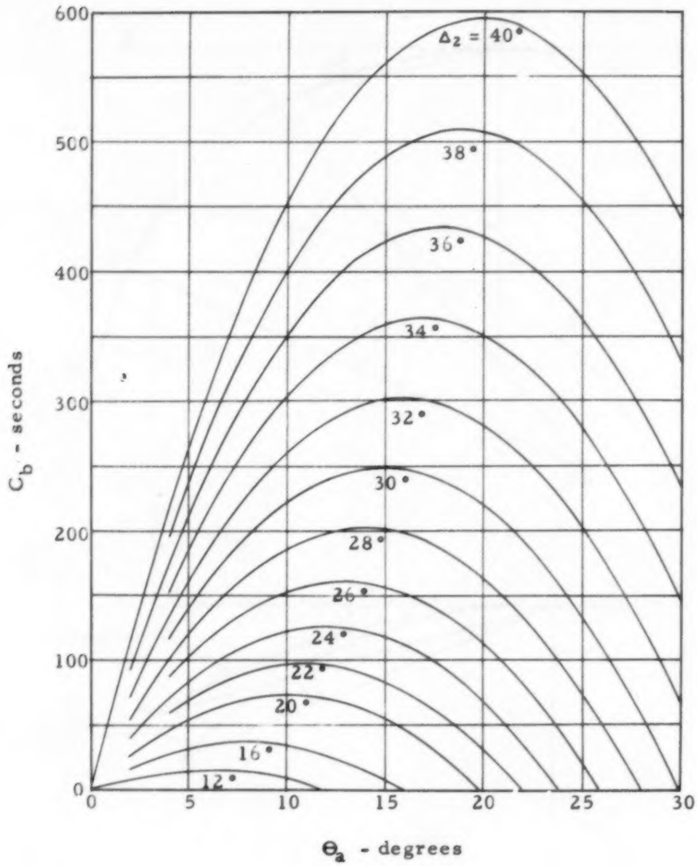


Fig. 3
Correction, C_b, to be added to
Δ_2 to obtain β

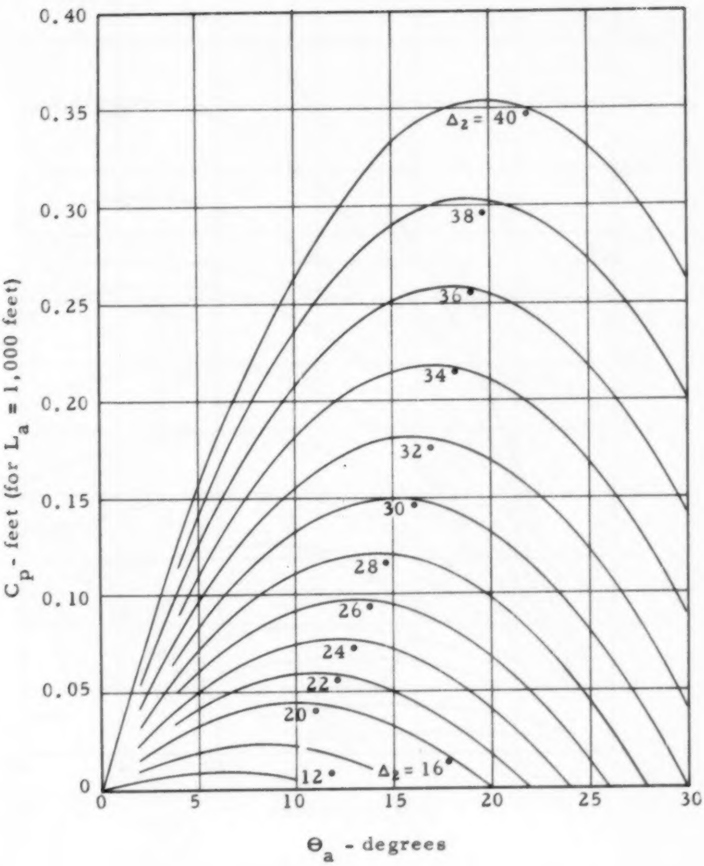


Fig. 4

Correction, C_p, for L_a = 1,000 feet, to be subtracted from p.

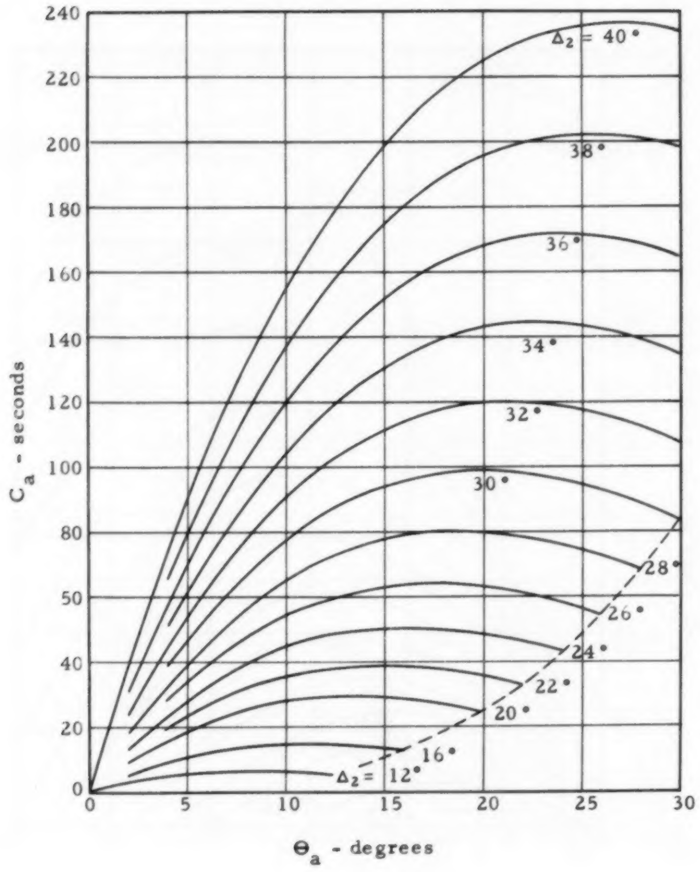


Fig. 5

Correction, C_a, to be added to nominal deflection angle when spiral layout is in direction of decreasing curvature and subtracted when spiral layout is in direction of increasing curvature.

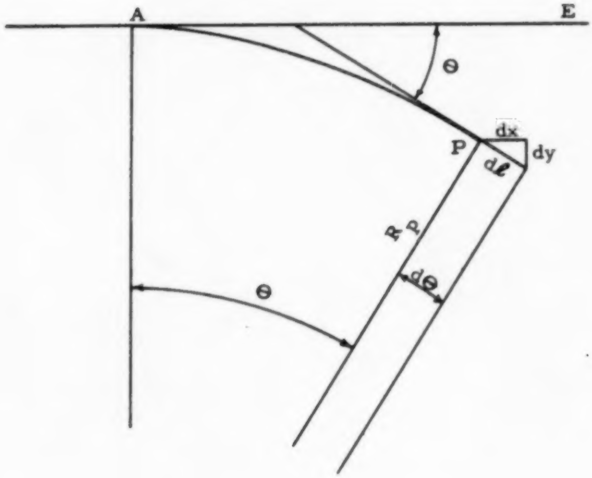


Fig. 6

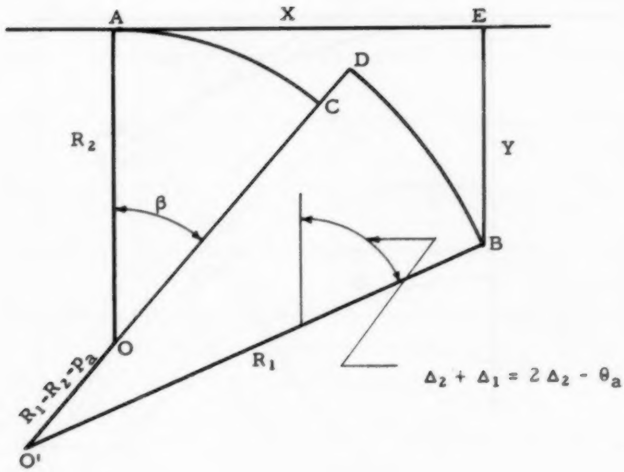


Fig. 7